Some mod p numerics on the homotopy groups of the groups of diffeomorphisms of the disc

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Here we will put some concrete values into a theorem in [BL74]. The theorems in this paper are derived from their proof of Morlets theorem in [BL], that is the homotopy equivalence $\mathrm{Diff}_\partial(D^n) \simeq \Omega^{n+1}\mathrm{PL}_n/\mathrm{O}_n$.

Theorem ([BL74], Thm 7.4)**.** *Let p be an odd prime, let $1 \leq r \leq p-2$ and $-2(p^2-p-2) \leq s < 2p + 2(r-1)(p^2-p-1) - 2$. Then there exists a subgroup*

$$\mathbb{Z}_p \subseteq \pi_{2p+2(r-1)(p^2-p-1)-2-s}\mathrm{Diff}_\partial(D^{2(p^2-p-2)+s})$$

We created the code below to genearte the pairs, intending to tabulate them. However they grow quickly and so we will provide the code and then some visualisations.

```
from sympy import sieve, prime, bernoulli
import math

n = 20 #The number of primes that you want.

sieve.extend_to_no(n) #Initialise the sieve to contain the primes I want
primes = [sieve[i] for i in range(2, n + 1)]

# initialise a dictionary where the keys are the elements of the list ‘primes‘
prime_dict = dict.fromkeys(primes)
# iterate through ‘primes‘ and assign the upper bounds for the s value.
for idx, p in enumerate(primes, start=1):
  prime_dict[p] = [2*p+2*(r-1)*(p**2-p-1)-2 for r in range(1,p-1)]

#Want a list (prime, homotopy group, dim of disc)
#this tells us that \Z_p \subseteq \pi_{homotopty group} Diff D^{dim of disc}
pairs = []
for p in primes:
    for r in prime_dict[p]:
        pairs = pairs + [(p, r - s, 2*(p**2-p-2)+s) for s in range (-2*(p**2 - p - 2),
     r)]
```

Listing 1: Code to generate triples (prime ho-index dim disc)

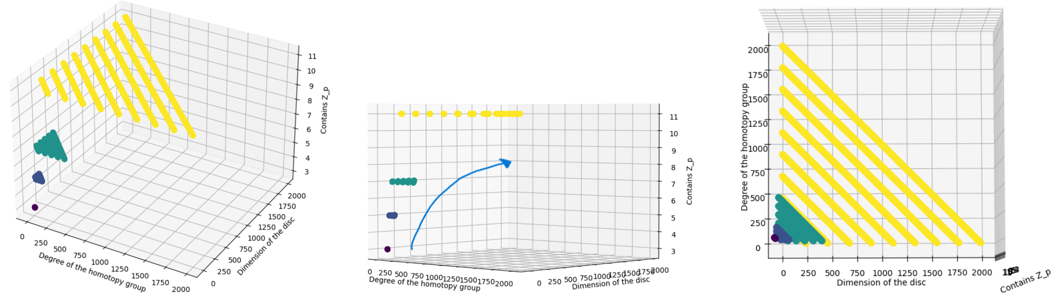

Figure 1: Middle shows the parabolic curve. Right shows that the last set of images is just projecting away the information of the prime.

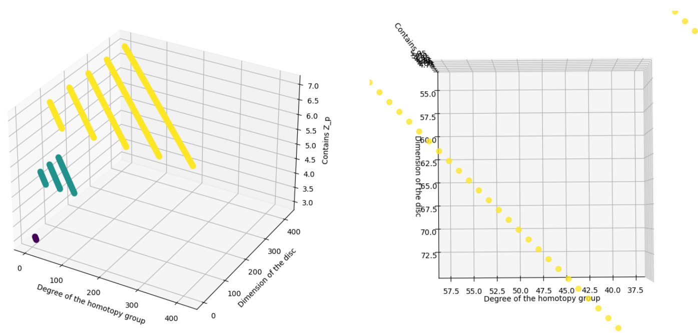

Figure 2: Increased resolution on the lower primes.

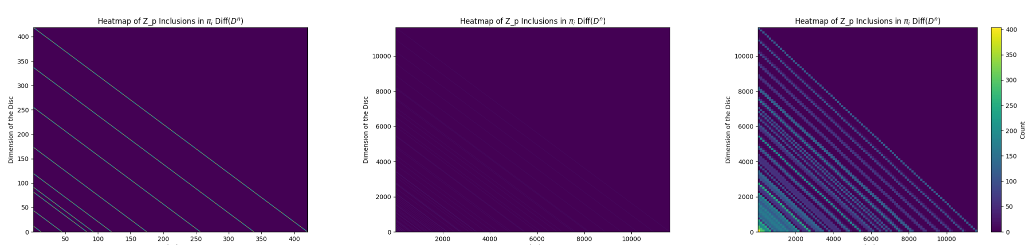

Figure 3: Forgetting the value of the primes. The middle and Right are for the first 8 primes and are very zoomed out. The Right graph has bigger bins, presumably as the primes increase the purple would fill out, but preserve this proportional spread.

References

[BL] Dan Burghelea and Richard Lashof. The homotopy type of the space of diffeomorphisms. I. 196(0):1–36.

[BL74] Dan Burghelea and Richard Lashof. The homotopy type of the space of diffeomorphisms. II. *Transactions of the American Mathematical Society*, 196(0):37–50, 1974.